Growth Models

Helicopter Tour

Eduardo Ley

THE WORLD BANK

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Outline

Harrod-Domar Model

Model with Excess Labor (1930s slump fears in Europe/US)

Assumptions

(A1) Output is proportional to Capital:

$$Y_t = \min\{aK_t, bL_t\} = aK_t$$

($v = 1/a$ is the capital-output ratio.)

(A2) Investment *ex ante* equals Saving:

$$I_t = S_t$$

(A3) Saving proportional to Output:

$$S_t = sY_t$$

Harrod-Domar Model

Mechanics

Change of output is proportional to the change in capital,

$$\Delta Y = a\Delta K \tag{1}$$

over time:

$$\frac{dY_t}{dt} = \dot{Y}_t = a\dot{K}_t \tag{2}$$

$$= aI_t = (as)Y_t \tag{3}$$

therefore the rate of Output Growth, g is given by

$$g = \frac{\dot{Y}_t}{Y_t} = as \tag{4}$$

Harrod-Domar Model

Knife-Edge Equilibrium

Additional Assumptions

(A4) Labor force increases at the rate n

(A5) Labor productivity increases at rate m

Now, in equilibrium, we must have

$$g = as = n + m \tag{5}$$

Knife-Edge Equilibrium

- If $g < n + m$, then we run out of jobs: unemployment
- If $g > n + m$, then we run out of workers

Harrod-Domar Model

Uses of model

$$g = as = n + m$$

Assess

- required g to achieve full employment
- required s to achieve a target g

Harrod-Domar Model

Issues

Issues

- $I = sY = S$
  - in advanced economies, savings and investment decisions independent of each other
  - in developing economies, S and I are interdependent. Increased saving depends more on opening up of investment opportunities (or removal of obstacles) than on increased income
- $Y = aK$, output-capital ratio
  - stable $a \in (0.2, 0.6)$ in developed economies (portfolio of projects with balanced distribution of a).
  - not so in developing countries, where also normal productivity of capital may held back by bottlenecks or shortages of complementary factors, and can jump up when these constrains are relaxed.

Harrod-Domar Model

From Harrod-Domar to Solow

Knife-Edge Equilibrium—need $g = n + m$

- Cannot expect to hold in general
- Need to make one of those an endogenous variable
- Solow makes a endogenous

$$a = \frac{Y}{K}$$

Solow model—substitution of capital for labor

- If labor becomes scarce, $n < sa - m$, then the wage rate increases and firms will substitute capital for labor: $\downarrow a$
- If labor becomes abundant, $n > sa - m$, then the wage rate decreases and firms will substitute labor for capital: $\uparrow a$

Solow's Growth Model

Setup

Model Setup

- Aggregate Output: $Y_t = F(K_t, A_t L_t)$
- Exogenous Technological Progress: $\dot{A}_t / A_t = m$
- Rate of Population Growth: $\dot{L}_t / L_t = n$
- Law of Motion of Capital Stock: $\dot{K}_t = I_t - \delta K_t$
- Savings-Investment Balance: $S_t = sY_t = I_t$

Main assumptions

(A1) Diminishing returns to K for a given state of technology

(A2) Exogenous technological progress, rate: m

(A3) All countries share the same technology

Solow's Growth Model

Rescale to units of effective labor

Assuming constant returns to scale (simplifying but not essential assumption):

$$F(\lambda K, \lambda AL) = \lambda F(K, AL) \tag{6}$$

make $\lambda = (A_t L_t)^{-1}$, and then, per-capita output:

$$y_t = \frac{Y_t}{A_t L_t} = \frac{F(K_t, A_t L_t)}{A_t L_t} = F\left(\frac{K_t}{A_t L_t}, 1\right) = f(k_t) \tag{7}$$

lower-case symbols, k_t and y_t, shall denote normalised quantities —i.e., measured in units of *effective labor*, $A_t L_t$.

Solow's Growth Model

Solution

- The accumulation equation for per capita capital, k_t, is given by:

$$\dot{k}_t = sf(k_t) - (n + m + \delta)k_t$$

- In the (unique, positive, and locally stable) steady-state equilibrium:

$$\dot{k}_* = 0 \quad \Rightarrow \quad sf(k_*) = (n + m + \delta)k_*$$

  so that gross investment must take care of effective-labor growth, $(n + m)$, and depreciation, δ.

Solow's Growth Model

Steady-state output per worker

$$k_* = \frac{sf(k_*)}{n + m + \delta}$$

- Steady state capital per worker depends
  - ⊕ positively on the saving (investment) rate, s,
  - ⊖ inversely on the effective-labour growth rate, $(n + m)$, and depreciation rate, δ,
- Similarly for ouput per effective worker

$$f(k_*) = f\left(\frac{sf(k_*)}{n + m + \delta}\right)$$

- Note: (s, n, δ) determine income levels, not growth rates, which are determined by the rate of technological progress (m).

Solow's Growth Model
Transition Dynamics

The *farther* away the economy is from its long run equilibrium the *faster* is the rate of growth of the capital stock and output.

$$k_t < k_* \Rightarrow \dot{k}_t > 0 \quad \text{and} \quad k_t > k_* \Rightarrow \dot{k}_t < 0$$
$$y \approx (k_* - k_t)[(n + m + \delta) - f'(k_t)] \propto (k_* - k_t)$$

Comparative Statics

Suppose that the savings rate s exogenously increases to $s' > s$

- New steady state has higher capital per worker and output per worker.
- There is a monotonic transition path from old to new steady state.

Solow's Growth Model
Assesment

1. Differences in income levels across countries explained in the model by differences in parameters: (s, n, m, δ).
2. Rich countries have higher saving (investment) rates relative to population growth than poor countries.
   - Changes in relative position: countries whose s moves up, relative to other countries, move up in income distribution. (Reverse with $(n + m)$.)
3. Cross-Country Variation in growth rates:
   - *Permanent* differences can only be due to differences in rate of technological progress m—if everyone has access to the same technology then growth rates must be the same.
   - *Temporary* differences are due to transition dynamics.
4. Variability of growth rates over time for a given country can be explained by transition dynamics and/or shocks to the parameters.

Solow's Growth Model

Policy Pessimism

Policy Implications

1. In the long run, growth is driven by technological progress, not by saving rate
2. Differences in per-capita GDP reflect differences in rates of saving and population growth
3. Cross-country convergence in growth rates
4. Cross-country conditional convergence in GDP levels

Endogenous Growth Models

Policy optimism—long run growth is affected by saving and other policies

Main assumptions

1. Diminishing returns to capital for given state of technology
2. Technological progress (productivity gains) arises from within the economic system
3. Countries use different technologies, owing to costly technology transfer

Implications

1. In the long run, growth is driven by technological progress, which is endogeneously determined
2. Differences in per-capita GDP can reflect differences in technology
3. Cross-country divergence in levels and growth rates of per-capita GDP

From Exogenous to Endogenous Growth

Common Model Setup

- Aggregate Output: $Y_t = F(K_t, A_t L_t)$
- Population Growth: $\dot{L}_t / L_t = n$
- Law of Motion of Capital Stock: $\dot{K}_t = I_t - \delta K_t$
- Savings-Investment Balance: $S_t = sY_t = I_t$

Technological Progress

- Solow's Model: $\dot{A}_t = mA_t$
- Romer's Model: $\dot{A}_t = \eta A_t^{\phi} L_{A,t}^{\lambda}$
- Generate productivity gains from within—e.g., investing in innovation, externalities associated with human capital

Harrod-Domar Model Solow Model Endogenous Growth Growth Accounting

Endogenous Growth

Romer's Model

Key change

$$\dot{A}_t = \eta A_t^{\phi} L_{A,t}^{\lambda}$$

- Labor is used for innovation, $L_{A,t}$, or production
- Rate of innovation depends on number of researchers and stock of knowledge
- $\phi > 0$ — productivity of research increases with stock of knowledge
- $\lambda > 1$ — implies positive spillovers

If a constant fraction of population is employed in R& D then all per-capita growth is due to technological progress, $g = g_A$.

Endogenous Growth

Romer's Model

Technological Progress

$$g_A = \frac{\dot{A}_t}{A_t} = \eta A_t^{(\phi-1)} L_{A,t}^{\lambda}$$

- Constant growth rate implies $g_A = \lambda n/(1-\phi)$
- Romer Mark I: $\lambda = \phi = 1$, so $g_A = \eta L_{A,t}$.
- Productivity of research grows over time as A (knowledge) accumulates
- At odds with US data — given increase in R&D, and that annual per capita growth in the US is less than 2%.

Endogenous Growth

AK Model

- Technological knowledge is a form of capital
- Technological progress is a form of saving
- Technological progress is advanced by
  - innovation (driven by prospect of monopoly rents)
  - implementation (driven by distance from technology frontier)

AK Model

$$\begin{aligned} Y_t &= A_t K_t \\ A_t &= \gamma A_{t-1}, \quad \gamma > 1 \\ \dot{K}_t &= (sA_t - \delta) K_t \\ g_Y &= g_K = (sA_t - \delta) \end{aligned}$$

Endogenous Growth

Aghion-Howitt Innovation-based Model

- Innovation is not the same thing as capital accumulation
  - Innovation involves change and obsolescence
  - Growth involves conflict between winners and losers
- Technology transfer is possible but costly
  - Two strategies for technological progress: innovation and implementation
  - Long run convergence in growth rates is possible through technology transfer
- Long run growth depends on policies and institutions that affect:
  1. The incentive to innovate
  2. The ability to foster and finance innovation
  3. The incentive and ability to oppose innovation

Solow Growth Accounting

Accounting for Growth Facts

- Growth models present a theoretical framework for understanding the sources of economic growth, and the consequences for long-run growth of changes in the economic environment and in economic policy.
- Often, however, we wish to examine economic growth in a more agnostic framework—without necessarily being bound to pre-adopt the conclusions of any given model.
- In order to conduct such analysis, economists have built up an alternative framework called growth accounting to obtain a factual perspective on the sources of economic growth.

Solow Growth Accounting

K, *L*, and the Solow Residual

US GDP Growth (1948–2001)

	1948–2001	1948–1973	1974–1995	1996–2001
GDP growth	2.5	3.3	1.5	2.5
$\oplus\ \Delta K$	0.9	0.9	0.7	1.2
$\oplus\ \Delta L$	0.2	0.2	0.2	0.4
$\oplus\ \Delta TFP$	1.3	2.1	0.9	0.9
ΔTFP as %	52%	64%	40%	36%

- Avoid over-interpretation, just get qualitative idea
- "No amount of (apparent) statistical evidence will make a statement invulnerable to common sense"
- Role of investment in spreading innovations

Harrod-Domar Model | Solow Model | Endogenous Growth | Growth Accounting

US 1929–1982: $g = 3.1$ percent per year

Growth Accounting by Edward Dennison

US 1929–1982: $g = 3.1$ percent per year			
i	Source	share	g_i
1	Δ constant-education labor	25%	0.8
2	Δ education	16%	0.5
3	Δ capital	12%	0.4
4	Improved allocation of resources	11%	0.3
5	Economies of scale	11%	0.3
6	Δ Technological progress	34%	1.1
7	∇ other stuff (Env Reg)	(9%)	(0.3)

(Source: R Solow's Nobel lecture)

Solow Growth Accounting
Setup

$$Y_t = F(K_t, L_t, A_t)$$

differentiate with respect to time:

$$\dot{Y}_t = F_K \dot{K}_t + F_L \dot{L}_t + F_A \dot{A}_t$$

divide by output and manipulate:

$$\begin{aligned}
\frac{\dot{Y}_t}{Y_t} &= \frac{F_K K_t}{Y_t}\frac{\dot{K}_t}{K_t} + \frac{F_L L_t}{Y_t}\frac{\dot{L}_t}{L_t} + \frac{F_A A_t}{Y_t}\frac{\dot{A}_t}{A_t} \\
g_Y &= \left(\frac{F_K K_t}{Y_t}\right) g_K + \left(\frac{F_L L_t}{Y_t}\right) g_L + \underbrace{\left(\frac{F_K A_t}{Y_t}\right) g_A}_{T\dot{F}P}
\end{aligned}$$

Solow Growth Accounting
Competitive Factor Markets

Further assume that capital and labor are traded in competitive markets and paid their marginal products, r and w:

$$g_Y = \underbrace{\left(\frac{rK_t}{Y_t}\right)}_{\sigma_K} g_K + \underbrace{\left(\frac{wL_t}{Y_t}\right)}_{\sigma_L} g_L + T\dot{F}P$$

where σ_K and σ_L are capital and labor shares in the national income.

Solow Growth Accounting

Competitive Factor Markets

Manipulating,

$$g_Y = \sigma_K g_K + \sigma_L g_L + T\dot{F}P$$

we obtain:

$$\begin{aligned} T\dot{F}P &= g_Y - [\sigma_K g_K + \sigma_L g_L] \\ &= g_Y - [\sigma_K g_K + (1-\sigma_K) g_L] \\ &= (g_Y - g_L) - \sigma_K (g_K - g_L) \\ &= g_y - \sigma_K g_k \end{aligned}$$

where $y = Y/L$ and $k = K/L$ are in per capita terms.

Solow Growth Accounting

Per capita productivity change

This expression can be turned around:

$$g_y = \sigma_K g_k + T\dot{F}P$$

and extended by, say, improvements in worker's quality, q:

$$g_y = \sigma_K g_k + \sigma_L q + T\dot{F}P$$

Where does TFP come from?

- Most people tend to associate TFP with the introduction of new technology.
- In fact it could be the result of an invention, the adoption of an existing technology; a managerial innovation; the re-allocation of factors across sectors and firms.
- The common feature is that in all cases the change results in a real cost reduction.

Perspiration *vs* Inspiration

Alwyn Young: The Myth of Asia's Miracle

Country	Time	g_Y	σ_K	$\sigma_K g_K$	$\sigma_L g_L$	$T\dot{F}P$
Germany	60-90	3.2	0.40	0.59	-0.08	0.49
Italy	60-90	4.1	0.38	0.49	0.03	0.48
UK	60-90	2.5	0.39	0.52	-0.04	0.52
Argentina	40-80	3.6	0.54	0.43	0.26	0.31
Brazil	40-80	6.4	0.45	0.51	0.20	0.29
Chile	40-80	3.8	0.52	0.34	0.26	0.40
Mexico	40-80	6.3	0.63	0.41	0.23	0.36
Japan	60-90	6.8	0.42	0.57	0.14	0.29
Hong Kong	66-90	7.3	0.37	0.42	0.28	0.30
Singapore	66-90	8.5	0.53	0.73	0.31	-0.04
South Korea	66-90	10.3	0.32	0.46	0.42	0.12
Taiwan	66-90	9.1	0.29	0.40	0.40	0.20

A Vision of the Growth Process

A Harberger AEA Presidential address

1. A small-to-modest fraction of industries can account for 100% of aggregate real cost reduction in a period;
2. The complementary fraction of industries contain winners and losers, the TFP contribution of which may cancel each other;
3. The losers are a very important part of the picture most of the time, and contribute greatly to the variations we observe in aggregate TFP performance; and
4. There is little evidence of persistence from period to period of the leaders in TFP performance.

Profiles of TFP Growth — USA

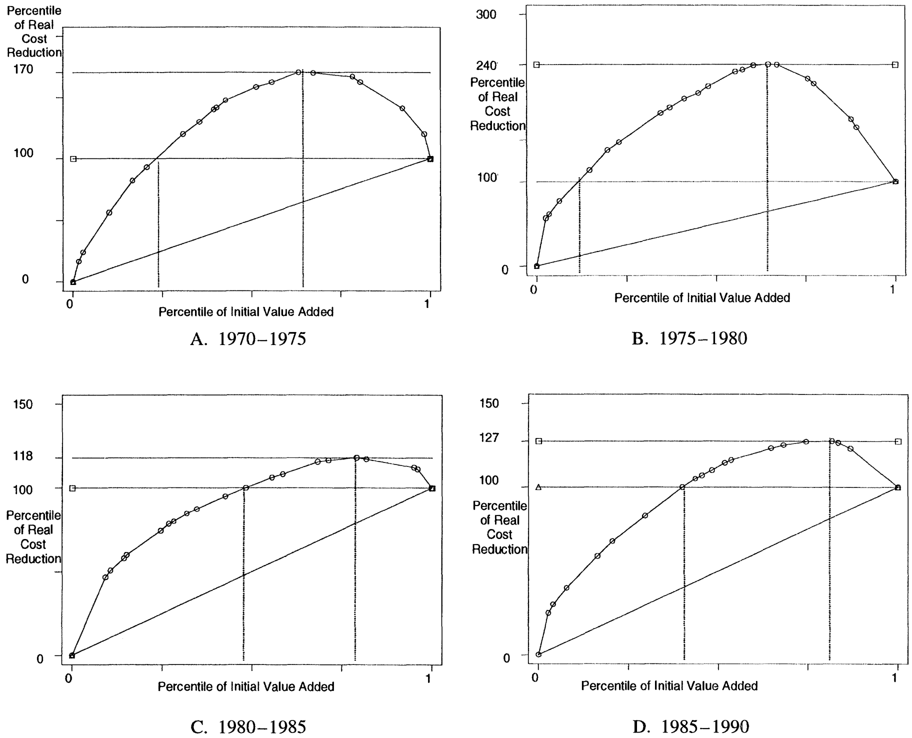

Figure 1. Profiles of TFP Growth Among U.S. Manufacturing Branches

Harrod-Domar Model Solow Model Endogenous Growth Growth Accounting

TFP Growth — Mexican Manufacturing

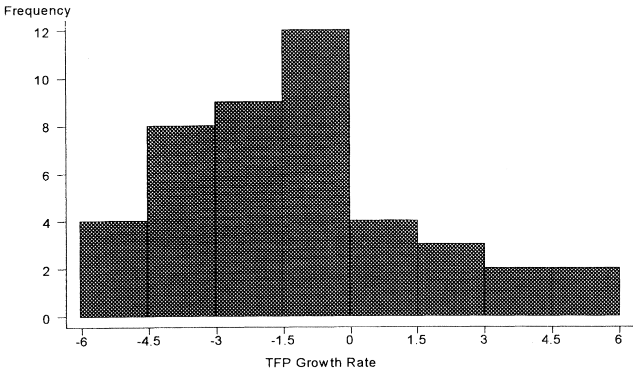

Figure 6A. Average Annual TFP Growth Rate: Mexican Manufacturing 1984–1994
Frequency Distribution, 44 Industrial Branches

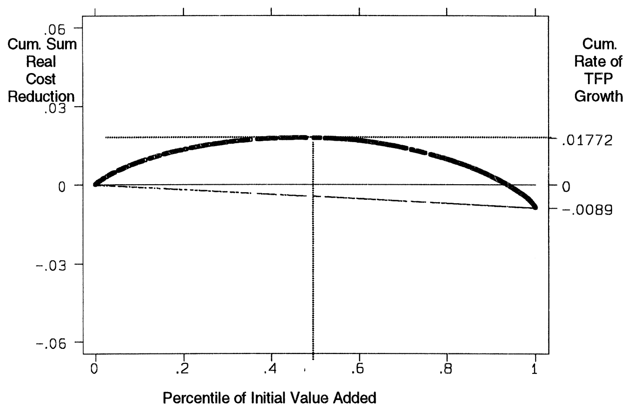

Figure 4. TFP Growth Profile in Mexican Manufacturing Sector (1892 Establishments, 1984–1994)

Negative productivity change

What does it mean? It means that the cost of production in these activities has increased more than the value of output. As a result these activities are transitorily or permanently unprofitable.

What causes it? It may be the result of an increase in costs of production which make the activity non-competitive, technological innovations that make products and/or technologies obsolete, changes in consumer's tastes, competition from imports etc.

Negative productivity change

Implications for income distribution and growth

No Pain, No Gain

- The faster resources are released from activities experiencing negative total factor productivity and reallocated to expanding sectors the higher is the impact of an innovation on growth.
- The effect of growth on income distribution depends on what happens in the job creation and in the job destruction end of the market.
- Often what happens in the contracting or 'losers' end of the market may be more important for income distribution and growth purposes than what happens at the expanding or 'winners' end.